SOLID STATE PHYSICS SOLUTIONS

GATE-2010

Q1. The valence electrons do not directly determine the following property of a metal

(a) Electrical conductivity (b) Thermal conductivity

(c) Shear modulus (d) Metallic luster

Ans: (c)

Q2. Consider *X*-ray diffraction from a crystal with a face-centered-cubic (*fcc*) lattice. The lattice plane for which there is NO diffraction peak is

(a) (2, 1, 2) (b) (1, 1, 1) (c) (2, 0, 0) (d) (3, 1, 1)

Ans: (a)

Q3. The Hall coefficient, R_H, of sodium depends on

(a) The effective charge carrier mass and carrier density

(b) The charge carrier density and relaxation time

(c) The charge carrier density only

(d) The effective charge carrier mass

Ans: (c)

Q4. The Bloch theorem states that within a crystal, the wavefunction, $\psi(\vec{r})$, of an electron has the form

(a) $\psi(\vec{r}) = u(\vec{r})e^{i\vec{k}.\vec{r}}$ where u($\vec{r}$) is an arbitrary function and $\vec{k}$ is an arbitrary vector

(b) $\psi(\vec{r}) = u(\vec{r})e^{i\vec{G}.\vec{r}}$ where u($\vec{r}$) is an arbitrary function and $\vec{G}$ is a reciprocal lattice vector

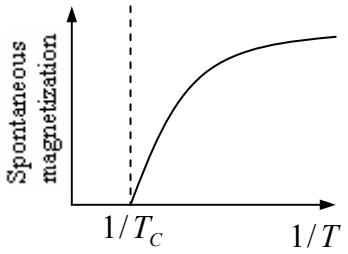
(c) $\psi(\vec{r}) = u(\vec{r})e^{i\vec{G}.\vec{r}}$ where $u(\vec{r}) = u(\vec{r} + \vec{\Lambda})$, $\vec{\Lambda}$ is a lattice vector and $\vec{G}$ is a reciprocal lattice vector

(d) $\psi(\vec{r}) = u(\vec{r})e^{i\vec{k}.\vec{r}}$ where $u(\vec{r}) = u(\vec{r} + \vec{\Lambda})$, $\vec{\Lambda}$ is a lattice vector and $\vec{k}$ is an arbitrary vector

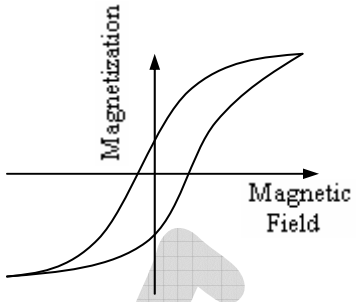
Ans: (d)

Q5. In an experiment involving a ferromagnetic medium, the following observations were made. Which one of the plots does NOT correctly represent the property of the medium? (T_C is the Curie temperature)

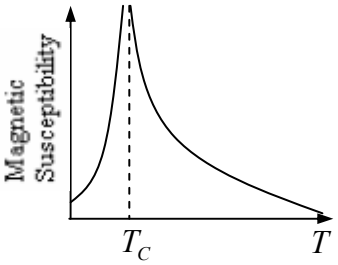
(a)



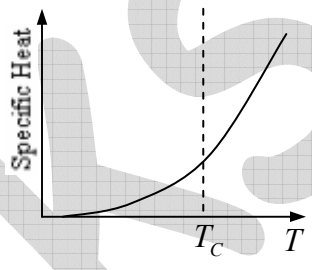
(b)



(c)



(d)



Ans: (c)

Q6. The thermal conductivity of a given material reduces when it undergoes a transition from its normal state to the superconducting state. The reason is:

(a) The Cooper pairs cannot transfer energy to the lattice

(b) Upon the formation of Cooper pairs, the lattice becomes less efficient in heat transfer

(c) The electrons in the normal state lose their ability to transfer heat because of their coupling to the Cooper pairs

(d) The heat capacity increases on transition to the superconducting state leading to a reduction in thermal conductivity

Ans: (d)

Q7. For a two-dimensional free electron gas, the electronic density n, and the Fermi energy E_F, are related by

(a) $n = \dfrac{(2mE_F)^{3/2}}{3\pi^2\hbar^3}$ (b) $n = \dfrac{mE_F}{\pi\hbar^2}$ (c) $n = \dfrac{mE_F}{2\pi\hbar^2}$ (d) $n = \dfrac{2^{1/3}(mE_F)^{1/3}}{\pi\hbar}$

Ans: (b)

Solution: For two dimensional gas, the number of possible k-states between k and $k+dk$ is

$g(k)dk = \left(\frac{L}{2\pi}\right)^2 2\pi k\, dk = 2\left(\frac{L}{2\pi}\right)^2 2\pi k\, dk$ it is multiplied by 2 for electron gas

Since $k^2 = \frac{2mE}{\hbar^2} \because 2k\, dk = \frac{2m}{\hbar^2} dE \Rightarrow 2\pi k\, dk = \frac{2\pi m}{\hbar^2} dE$

$\therefore\ g(E)dE = 2\left(\frac{L}{2\pi}\right)^2 \cdot \frac{2\pi m}{\hbar^2} dE$

The total number of electrons at $T = 0^0 K$ is

$$N = \int_0^{E_F} g(E)dE \times F(E) = \int_0^{E_F} g(E)dE = 2\pi \cdot \frac{2m}{\hbar^2}\left(\frac{L}{2\pi}\right)^2 \int_0^{E_F} dE = 2\pi \cdot \frac{2m}{\hbar^2} \cdot \frac{L^2}{4\pi^2} \cdot E_F$$

$$\Rightarrow N = \frac{m}{\pi\hbar^2} \cdot L^2 E_F \Rightarrow E_F = \frac{\pi\hbar^2}{m}\left(\frac{N}{L^2}\right) = \frac{\pi\hbar^2}{m} \cdot n \Rightarrow n = \frac{mE_F}{\pi\hbar^2}$$

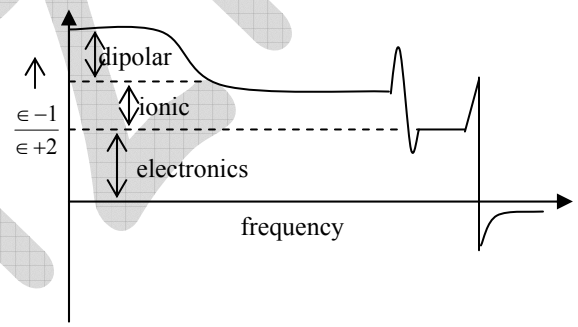
Q8. Far away from any of the resonance frequencies of a medium, the real part of the dielectric permittivity is

(a) Always independent of frequency (b) Monotonically decreasing with frequency

(c) Monotonically increasing with frequency (d) A non-monotonic function of frequency

Ans: (a)

Solution:



GATE-2011

Q9. The temperature (T) dependence of magnetic susceptibility (χ) of a ferromagnetic substance with a Curie temperature (T_c) is given by

(a) $\frac{C}{T-T_c}$, for $T < T_c$ (b) $\frac{C}{T-T_c}$, for $T > T_c$

(c) $\frac{C}{T+T_c}$, for $T > T_c$ (d) $\frac{C}{T+T_c}$, for all temperatures

where C is constant .

Ans: (b)

Q10. The order of magnitude of the energy gap of a typical superconductor is

(a) 1 MeV (b) 1 KeV (c) 1 eV (d) 1 meV

Ans: (d)

Q11. For a three-dimensional crystal having N primitive unit cells with a basis of p atoms, the number of optical branches is

(a) 3 (b) $3p$ (c) $3p-3$ (d) $3N-3p$

Ans: (c)

Q12. For an intrinsic semiconductor, m_e^* and m_h^* are respectively the effective masses of electrons and holes near the corresponding band edges. At a finite temperature the position of the Fermi level

(a) depends on m_e^* but not on m_h^* (b) depends on m_h^* but not on m_e^*

(c) depends on both m_e^* and m_h^* (d) depends neither on m_e^* nor on m_h^*

Ans: (c)

Solution: The Fermi level for intrinsic semicondutor is $E_F = \frac{E_c + E_v}{2} + \frac{3}{4}k_B T \ln\left(\frac{m_h^*}{m_e^*}\right)$

Q13. A metal with body centered cubic (bcc) structure show the first (i.e. smallest angle) diffraction peak at a Bragg angle of $\theta = 30^o$. The wavelength of X-ray used is 2.1 Å. The volume of the PRIMITIVE unit cell of the metal is

(a) 26.2 $(Å)^3$ (b) 13.1$(Å)^3$ (c) 9.3 $(Å)^3$ (d) 4.6 $(Å)^3$

Ans: (b)

Solution: According to Bragg's law $2d\sin\theta = \lambda$ where $d = \dfrac{a}{\sqrt{h^2+k^2+l^2}}$

For BCC structure the first diffraction peak appear for (110) plane.

$$\because d = \frac{a}{\sqrt{2}} \Rightarrow \frac{2a}{\sqrt{2}}\sin 30^0 = \lambda \Rightarrow \sqrt{2}a\sin 30^0 = 2.1A^0$$

$$\Rightarrow \sqrt{2}\,a\times\frac{1}{2} = 2.1A^0 \Rightarrow a = \sqrt{2}\times 2.1A^0 \Rightarrow a = 2.97A^0.$$

The volume primitive unit cell of BCC is volume $= \dfrac{a^3}{2} = \dfrac{26.2}{2}\left(A^0\right)^3 = 13.1\left(A^0\right)^3$

Common Data for Questions 14 and 15:

The tight binding energy dispersion (E-k) relation for electrons in a one-dimensional array of atoms having lattice constant a and total length L is

$$E = E_0 - \beta - 2\gamma\, cos\,(ka),$$

where E_0, β and γ are constants and k is the wave vector.

Q14. The density of states of electrons (including spin degeneracy) in the band is given by

(a) $\dfrac{L}{\pi\gamma\, a\sin(ka)}$ (b) $\dfrac{L}{2\pi\gamma\, a\sin(ka)}$ (c) $\dfrac{L}{2\pi\gamma\, a\cos(ka)}$ (d) $\dfrac{L}{\pi\gamma\, a\cos(ka)}$

Ans: (a)

Solution: $D(E) = 2\times 2\left(\dfrac{L}{2\pi}\right)\cdot\dfrac{1}{dE/dk} = 2\left(\dfrac{L}{2\pi}\right)\cdot\dfrac{2\times 1}{2a\gamma\sin(ka)} = \dfrac{2\times L}{2\pi\gamma a\sin(ka)}$

Q15. The effective mass of electrons in the band is given by

(a) $\dfrac{\hbar^2}{\gamma\, a^2\cos(ka)}$ (b) $\dfrac{\hbar^2}{2\gamma\, a^2\cos(ka)}$ (c) $\dfrac{\hbar^2}{\gamma\, a^2\sin(ka)}$ (d) $\dfrac{\hbar^2}{2\gamma\, a^2\sin(ka)}$

Ans: (b)

Solution: Effective mass $m^* = \dfrac{\hbar^2}{\left(\dfrac{d^2E}{dk^2}\right)} = \dfrac{\hbar^2}{2a^2\gamma\cos(ka)} = \dfrac{\hbar^2}{2\gamma a^2\cos(ka)}$

GATE-2012

Q16. For an ideal Fermi gas in three dimensions, the electron velocity V_F at the Fermi surface is related to electron concentration n as,

(a) $V_F \propto n^{2/3}$ (b) $V_F \propto n$ (c) $V_F \propto n^{1/2}$ (d) $V_F \propto n^{1/3}$

Ans: (d)

Solution: $V_F = \frac{\hbar}{m}\left(3\pi^2 n\right)^{1/3}$

Q17. The total energy, E of an ideal non-relativistic Fermi gas in three dimensions is given by $E \propto \frac{N^{5/3}}{V^{2/3}}$ where N is the number of particles and V is the volume of the gas. Identify the **CORRECT** equation of state (P being the pressure),

(a) $PV = \frac{1}{3}E$ (b) $PV = \frac{2}{3}E$ (c) $PV = E$ (d) $PV = \frac{5}{3}E$

Ans: (b)

Q18. Which one of the following CANNOT be explained by considering a harmonic approximation for the lattice vibrations in solids?

(a) Deby's T^3 law (b) Dulong Petit's law

(c) Optical branches in lattices (d) Thermal expansion

Ans: (d)

Solution: Thermal expansion in solid can only be explained if solid behave as a anharmonic oscillator.

Q19. A simple cubic crystal with lattice parameter a_c undergoes transition into a tetragonal structure with lattice parameters $a_t = b_t = \sqrt{2}a_c$ and $c_t = 2a_c$, below a certain temperature. The ratio of the interplanar spacing of (1 0 1) planes for the cubic and the tetragonal structure is

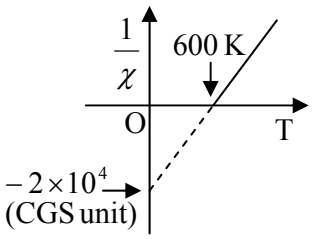
(a) $\sqrt{\frac{1}{6}}$ (b) $\frac{1}{6}$ (c) $\sqrt{\frac{3}{8}}$ (d) $\frac{3}{8}$

Ans: (c)

Solution: For Cubic Lattice $d_c = \frac{a}{\sqrt{h^2+k^2+l^2}} = \frac{a_c}{\sqrt{2}}$

For Tetragonal lattice $d_t = \frac{a}{\sqrt{\frac{h^2+k^2}{a^2}+\frac{l^2}{c^2}}} = \frac{2a_c}{\sqrt{3}}$. Therefore, the ratio is $\frac{d_c}{d_t} = \sqrt{\frac{3}{8}}$

Q20. Inverse susceptibility ($1/\chi$) as a function of temperature, T for a material undergoing paramagnetic to ferromagnetic transition is given in the figure, where O is the origin. The values of the Curie constant, C, and the Weiss molecular field constant, λ, in CGS units, are



(a) $C = 5\times10^{-5}, \lambda = 3\times10^{-2}$ (b) $C = 3\times10^{-2}, \lambda = 5\times10^{-5}$

(c) $C = 3\times10^{-2}, \lambda = 2\times10^{4}$ (d) $C = 2\times10^{4}, \lambda = 3\times10^{-2}$

Ans: (c)

Solution: $\frac{1}{\chi} = \frac{T - T_C}{C}$ and $T_C = \lambda C$. Here $T_C = 600K$ and $\frac{1}{\chi} = -2\times10^4$

Thus $C = 3\times10^{-2}$ and $\lambda = 2\times10^4$.

Common Data for Questions 21–22

The dispersion relation for a one dimensional monoatomic crystal with lattice spacing a, which interacts nearest neighbour harmonic potential is given by

$$\omega = A\left|\sin\frac{Ka}{2}\right|$$

where A is a constant of appropriate unit.

Q21. The group velocity at the boundary of the first Brillouin zone is

(a) 0 (b) 1 (c) $\sqrt{\frac{Aa^2}{2}}$ (d) $\frac{1}{2}\sqrt{\frac{Aa^2}{2}}$

Ans: (a)

Solution: At the first Brillouin zone the frequency is maximum and the group velocity which is the derivative of the angular frequency is zero.

Q22. The force constant between the nearest neighbour of the lattice is (M is the mass of the atom)

(a) $\frac{MA^2}{4}$ (b) $\frac{MA^2}{2}$ (c) MA^2 (d) $2MA^2$

Ans: (a)

Solution: $\because \omega = \sqrt{\frac{4\beta}{M}}\sin\left(\frac{ka}{2}\right) \Rightarrow A = \sqrt{\frac{4\beta}{M}} \Rightarrow \beta = \frac{MA^2}{4}$

GATE-2013

Q23. A phosphorous doped silicon semiconductor (doping density: $10^{17}/cm^3$) is heated from 100°C to 200°C. Which one of the following statements is CORRECT?

(a) Position of Fermi level moves towards conduction band

(b) Position of dopant level moves towards conduction band

(c) Position of Fermi level moves towards middle of energy gap

(d) Position of dopant level moves towards middle of energy gap

Ans: (c)

Solution: Phosphorous doped silicon semiconductors behave as a n-type semiconductor. In n-type semiconductor Fermi level lies near conduction band and moves toward middle of the band gap upon heating. At a very high temperature the Fermi level is near the middle of the band gap and semiconductor behaves as intrinsic semiconductor.

Q24. Considering the BCS theory of superconductors, which one of the following statements is **NOT CORRECT**? (h is the Plank's constant and e is the electronic charge)

(a) Presence of energy gap at temperature below the critical temperature

(b) Different critical temperature for isotopes

(c) Quantization of magnetic flux in superconduction ring in the unit of $\left(\frac{h}{e}\right)$

(d) Presence of Meissner effect

Ans: (c)

Solution: Quantization of magnetic flux in superconduction ring in the unit of $\left(\frac{h}{2e}\right)$

Q25. Group I contains elementary excitations in solids. Group II gives the associated field with these excitations. MATCH the excitations with their associated field and select your answer as per codes given below.

Group I	**Group II**
(P) phonon	(i) photon + lattice vibration
(Q) plasmon	(ii) electron +elastic deformation
(R) polaron	(iii) collective electron oscillations
(S) polariton	(iv) elastic wave

Codes

(a) $(P-iv),(Q-iii),(R-i),(S-ii)$ (b) $(P-iv),(Q-iii),(R-ii),(S-i)$

(c) $(P-i),(Q-iii),(R-ii),(S-iv)$ (d) $(P-iii),(Q-iv),(R-ii),(S-i)$

Ans: (b)

Solution: **Phonon**: Quantum of energy of the elastic wave in solid, produced due to the vibration of atoms in solid.

Plasmon: Quantum of energy of the wave produced due to the oscillation of plasma, which contains charged particles (positive ions and negative electrons or ions).

Polaron: A charge placed in a polarizable medium will be screened. The induced polarization will follow the charge carrier when it is moving through the medium. The carrier together with the induced polarization is considered as one entity, which is called a polaron.

Polariton: A polariton is a quasiparticle resulting from the mixing of a photon with phonon.

Q26. A lattice has the following primitive vector $(in\ \text{Å})$: $\vec{a} = 2(\hat{j} + \hat{k})$, $\vec{b} = 2(\hat{k} + \hat{i})$, $\vec{c} = 2(\hat{i} + \hat{j})$. The reciprocal lattice corresponding to the above lattice is

(a) BCC lattice with cube edge of $\left(\frac{\pi}{2}\right)\text{Å}^{-1}$

(b) BCC lattice with cube edge of $(2\pi)\text{Å}^{-1}$

(c) FCC lattice with cube edge of $\left(\frac{\pi}{2}\right)\text{Å}^{-1}$

(d) FCC lattice with cube edge of $(2\pi)\text{Å}^{-1}$

Ans: (a)

Solution: The reciprocal lattice vectors are

$$\vec{a}* = 2\pi\frac{\vec{b}\times\vec{c}}{\vec{a}\cdot(\vec{b}\times\vec{c})} = \frac{\pi}{2}\left(-\hat{i}+\hat{j}+\hat{k}\right)\text{Å}^{-1}$$

$$\vec{b}* = 2\pi\frac{\vec{c}\times\vec{a}}{\vec{a}\cdot(\vec{b}\times\vec{c})} = \frac{\pi}{2}\left(\hat{i}-\hat{j}+\hat{k}\right)\text{Å}^{-1}$$

$$\vec{c}* = 2\pi\frac{\vec{a}\times\vec{b}}{\vec{a}\cdot(\vec{b}\times\vec{c})} = \frac{\pi}{2}\left(\hat{i}+\hat{j}-\hat{k}\right)\text{Å}^{-1}$$

Q27. The total energy of an ionic solid is given by an expression $E = -\frac{\alpha e^2}{4\pi\varepsilon_0 r} + \frac{B}{r^9}$ where α is Madelung constant, r is the distance between the nearest neighbours in the crystal and B is a constant. If r_0 is the equilibrium separation between the nearest neighbours then the value of B is

(a) $\frac{\alpha e^2 r_0^8}{36\pi\varepsilon_0}$ (b) $\frac{\alpha e^2 r_0^8}{4\pi\varepsilon_0}$ (c) $\frac{2\alpha e^2 r_0^{10}}{9\pi\varepsilon_0}$ (d) $\frac{\alpha e^2 r_0^{10}}{36\pi\varepsilon_0}$

Ans: (a)

Solution: At $r = r_0$, $\left.\frac{dE}{dr}\right|_{r=r_0} = 0 = \frac{\alpha e^2}{4\pi\varepsilon_0 r_0^2} - \frac{9B}{r_0^{10}} \Rightarrow B = \frac{\alpha e^2 r_0^8}{36\pi\varepsilon_0}$

GATE-2014

Q28. The Miller indices of a plane passing through the three points having coordinates (0, 0, 1) $(1,0,0)\left(\frac{1}{2},\frac{1}{2},\frac{1}{4}\right)$ are

(a) (212) (b) (111) (c) (121) (d) (211)

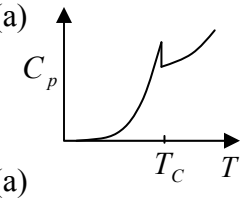
Ans: (a)

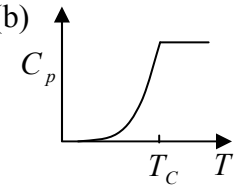
Solution: The equation of plane is determined from following determinant:

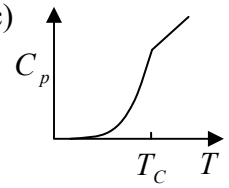
$$\begin{vmatrix} x-1 & y-0 & z-0 \\ -1 & 0 & 1 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{4} \end{vmatrix} = 0 \Rightarrow (x-1)\left(-\frac{1}{2}\right) - y\left(\frac{-1}{4}+\frac{1}{2}\right) + (z-1)\left(-\frac{1}{2}\right) = 0$$

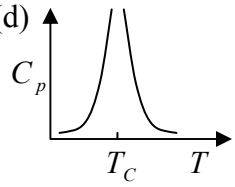
$\Rightarrow \frac{-x}{2} - \frac{y}{4} - \frac{z}{2} + \frac{1}{2} = 0 \Rightarrow 2x + y + 2z - 2 = 0$, $hx + ky + lz - 2 = 0$. Miller indices are $(2\,1\,2)$

Q29. The plot of specifies heat versus temperature across the superconducting transition temperature (T_c) is most appropriately represented by

(a) 

(b) 

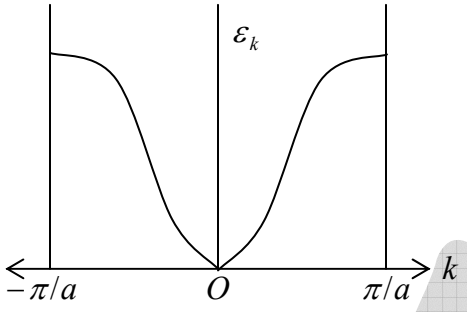
(c) 

(d) 

Ans: (a)

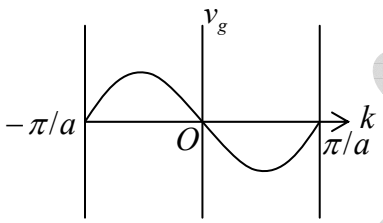
Solution: $C_V \propto e^{-\left(\frac{\Delta}{2kT}\right)}$

Q30. The energy ε_k for band electrons as a function of the wave vector k in the first Brillouin zone $\left(-\frac{\pi}{a} \le k \le \frac{\pi}{a}\right)$ of a one dimensional monoatomic lattice is shown as (a is lattice constant)

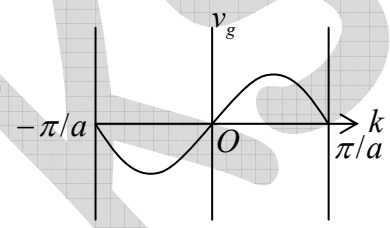


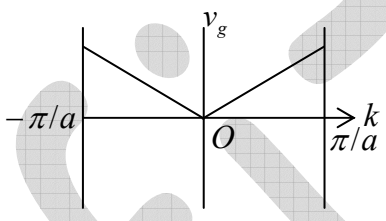
The variation of the group velocity v_g is most appropriately represented by

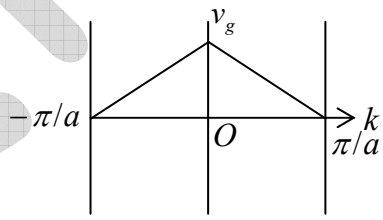
(a)



(b)







Ans: (b)

Solution: $E = \left(E_0 - \gamma\beta\left(\cos ka\right)\right)$

$$V_g = \frac{1}{\hbar}\frac{dE}{dk} = \frac{a\gamma\beta}{\hbar}\sin ka$$

Q31. For Nickel the number density is $8\times10^{23}\ atoms/cm^3$ and electronic configuration is $1s^2 2s^2 2p^6 3s^2 3p^6 3d^8 4s^2$. The value of the saturation magnetization of Nickel in its ferromagnetic state is _____$\times10^9\ A/m$.

(Given the value of Bohr magneton $\mu_B = 9.21\times10^{-21}\ Am^2$)

Ans: 4.42

Solution: Component of magnetic dipoles in a solid material are in the direction of external field.

$M_S =$ (Magnetic dipole moment) $\mu_B N = 0.6 \times 9.21 \times 10^{-21} \left(Am^2\right) N$.

(For N_i : *magnetic dipole moment* $= 0.6$, Fe :2.22, *For* Cu :1.2)

$N = \dfrac{\rho N_A}{A_N} = 8 \times 10^{29} / m^3$, $\mu_B = 9.21 \times 10^{-21} A/m$

$M_S = 0.6 \times 9.21 \times 10^{-21} \times 8 \times 10^{29} = 4.42 \times 10^9 \ A/m$, $A_n \rightarrow$ atomic weight

GATE-2015

Q31. The energy dependence of the density of states for a two dimensional non-relativistic electron gas is given by, $g(E) = CE^n$, where C is constant. The value of n is_____

Ans.: 0

Solution: We know that

$g(E) \propto E^{1/2}$ for $3-D$, $g(E) \propto E^0$ for $2-D$, $g(E) \propto E^{-1/2}$ for $1-D$

$\Rightarrow n = 0$ for $2-D$

Q32. The lattice parameters a, b, c of an orthorhombic crystal are related by $a = 2b = 3c$. In units of a the interplanar separation between the (110) planes is _____. (Upto three decimal places)

Ans.: 0.447

Solution: $d_{hkl} = \dfrac{1}{\sqrt{\dfrac{h^2}{a^2} + \dfrac{k^2}{b^2} + \dfrac{l^2}{c^2}}} \Rightarrow d_{110} = \dfrac{1}{\sqrt{\dfrac{1}{a^2} + \dfrac{1}{\left(\dfrac{a}{2}\right)^2} + 0}} = \dfrac{a}{\sqrt{5}} = 0.447 \quad \because a = 2b = 3c$

Q33. The dispersion relation for phonons in a one dimensional monoatomic Bravais lattice with lattice spacing a and consisting of ions of masses M is given by $\omega(k) = \sqrt{\dfrac{2c}{M}\left[1 - \cos(ka)\right]}$, where ω is the frequency of oscillation, k is the wavevector and C is the spring constant. For the long wavelength modes $(\lambda >> a)$, the ratio of the phase velocity to the group velocity is_____

Ans.: 1

Solution: $\omega(k) = \sqrt{\dfrac{2C}{M}\left[1 - \cos(ka)\right]}$

For long wavelength modes $(\lambda >> a)$

$$\because \cos(ka) \cong 1-\frac{(ka)^2}{2} \Rightarrow \omega(k) = \sqrt{\frac{2C}{M}\left[1-1+\frac{(ka)^2}{2}\right]} = a\sqrt{\frac{C}{M}}k$$

Phase velocity $v_P = \frac{\omega}{k} = a\sqrt{\frac{C}{M}}$ and Group velocity $v_g = \frac{d\omega}{dk} = a\sqrt{\frac{C}{M}} \Rightarrow \frac{v_P}{v_g} = 1$

Q34. In a Hall effect experiment, the hall voltage for an intrinsic semiconductor is negative. This is because (symbols carry usual meaning)

(a) $n \approx p$ (b) $n > p$ (c) $\mu_n > \mu_h$ (d) $m_h^* > m_n^*$

Ans.: (c)

Solution: The Hall voltage is $V_H = R_H JB$

where J : current density, B : magnetic field and R_H : Hall constant

$$R_H = \frac{1}{e}\frac{p\mu_p^2 - n\mu_n^2 + (p-n)\mu_n^2\mu_p^2B^2}{(n\mu_n + p\mu_p)^2 + (p-n)^2\mu_n^2\mu_p^2B^2}$$

For intrinsic semiconductor $(n = p = n_i)$ $R_H = \frac{1}{en_i}\frac{\mu_p - \mu_n}{\mu_p + \mu_n}$

In Intrinsic semiconductor $\mu_n > \mu_p$, therefore Hall voltage is negative.

Q35. Given that the Fermi energy of gold is 5.54 eV, the number density of electrons is _____ $\times 10^{28} m^{-3}$ (upto one decimal place)

(Mass of electron $= 9.11\times10^{-31}$ kg; $h = 6.626\times10^{-34}$ $j\cdot s$; $1\ eV = 1.6\times10^{-19}$ j)

Ans.: 5.9

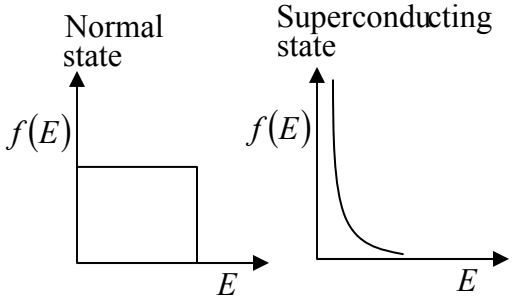
Solution: Relation between electron density (n) and Fermi energy (E_F) is

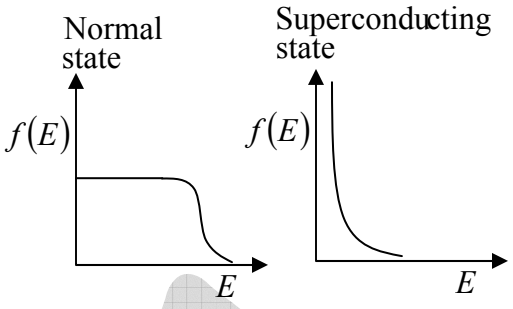
$$E_F = \frac{\hbar^2}{2m}(3\pi^2 n)^{2/3} \Rightarrow n = \frac{1}{3\pi^2}\frac{(2m)^{3/2}}{\hbar^3}\cdot E_F^{3/2}$$

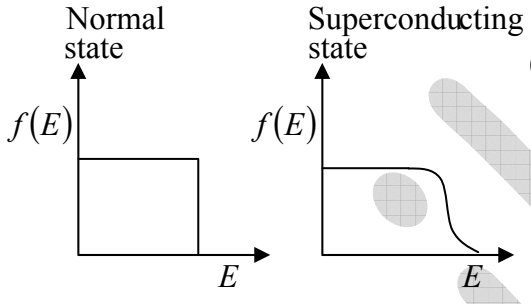
$$\Rightarrow n = \frac{1}{3\times(3.14)^2}\times\frac{\left(2\times9.1\times10^{-31}kg\right)^{3/2}}{\left(1.0546\times10^{-34}J\text{-}s\right)^3}\times\left(5.54\times1.6\times10^{-19}\ \ J\right)^{3/2}$$

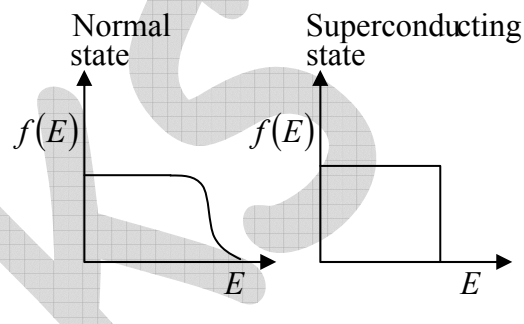
$$\Rightarrow n = \frac{1}{29.61}\times\frac{2.45\times10^{-45}\times8.35\times10^{-28}}{1.17\times10^{-102}}m^{-3} = 0.59\times10^{29}\ m^{-3} \Rightarrow n = 5.9\times10^{28}\ \ m^{-3}$$

Q36. Which one of the following represents the electron occupancy for a superconductor in its normal and superconducting states?

(a) 

(b) 

(c) 

(d) 

Ans. : (d)

Solution: In normal slide, some states below Fermi levels are empty and equal number of states above Fermi levels are filled. If material converts into a superconductor, electrons above the Fermi Level makes cooper pair and they fall back below level Fermi level as same energy released during cooper pair formation. Therefore, correct option is (d).

GATE-2016

Q37. Consider a metal which obeys the Sommerfield model exactly. If E_F is the Fermi energy of the metal at $T = 0K$ and R_H is its Hall coefficient, which of the following statements is correct?

(a) $R_H \propto E_F^{\frac{3}{2}}$ (b) $R_H \propto E_F^{\frac{2}{3}}$

(c) $R_H \propto E_F^{\frac{-3}{2}}$ (d) R_H is independent of E_F.

Ans.: (c)

Solution: $R_H = \frac{1}{ne}$, where $E_F = \frac{\hbar^2}{2m}\left(3\pi^2 n\right)^{2/3} \Rightarrow n = \left(\frac{2m}{\hbar^2}\right)^{3/2} \cdot \frac{\left(E_F\right)^{3/2}}{3\pi^2} \Rightarrow R_H \propto E_F^{-3/2}$

Q38. A one-dimensional linear chain of atoms contains two types of atoms of masses m_1 and m_2 (where $m_2 > m_1$), arranged alternately. The distance between successive atoms is the same. Assume that the harmonic approximation is valid. At the first Brillouin zone boundary, which of the following statements is correct?

(a) The atoms of mass m_2 are at rest in the optical mode, while they vibrate in the acoustical mode.

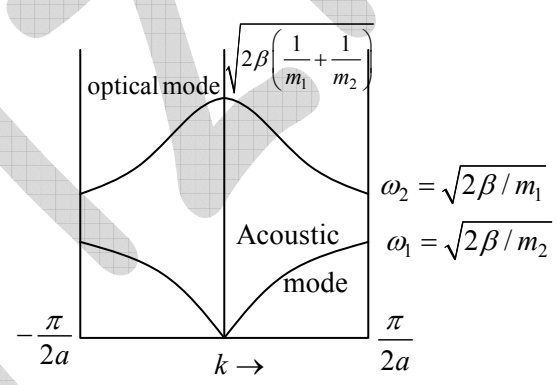
(b The atoms of mass m_1 are at rest in the optical mode, while they vibrate in the acoustical mode.

(c) Both types of atoms vibrate with equal amplitudes in the optical as well as in the acoustical modes.

(d) Both types of atoms vibrate, but with unequal, non-zero amplitudes in the optical as well as in the acoustical modes.

Ans.: (a)

Solution: In optical mode, at Brillouin zone boundary atom of heavier mass (m_2) is at rest, whereas in Acoustic mode, atoms of lighter mass (m_1) is at rest.



Q39. A solid material is found to have a temperature independent magnetic susceptibility, $\chi = C$. Which of the following statements is correct?

(a) If C is positive, the material is a diamagnet.

(b) If C is positive, the material is a ferromagnet.

(c) If C is negative, the material could be a type I superconductor.

(d) If C is positive, the material could be a type I superconductor.

Ans.: (b)

Solution: Susceptibility is defined as $\chi = \frac{\mu}{\mu_0} - 1$, where μ and μ_0 are permeability of medium and vacuum respectively.

(i) For Diamagnet; $\mu < \mu_0$, thus $\chi < 0$ i.e. χ is negative

(ii) For Ferromagnet, $\mu >> \mu_0$, thus $\chi >> 0$ i.e. χ is positive

(iii) For superconductor, $\mu = 0$, thus $\chi = -1$

Thus best answer is (b)

Q40. Atoms, which can be assumed to be hard spheres of radius R, are arranged in an *fcc* lattice with lattice constant a, such that each atom touches its nearest neighbours. Take the center of one of the atoms as the origin. Another atom of radius r (assumed to be hard sphere) is to be accommodated at a position $\left(0, \frac{a}{2}, 0\right)$ without distorting the lattice. The maximum value of $\frac{r}{R}$ is _____. (Give your answer upto two decimal places)

Ans.: 0.41

Solution: The new atom location is $\left(0, \frac{a}{2}, 0\right)$ i.e. it is on the middle of y - axis.

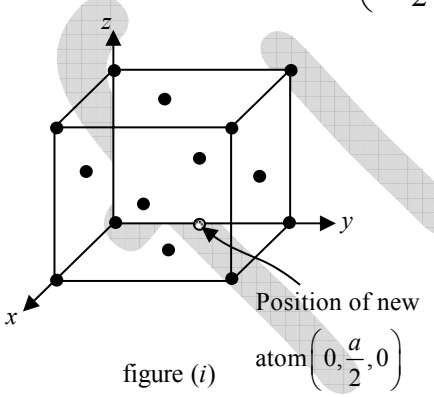


figure (*i*)

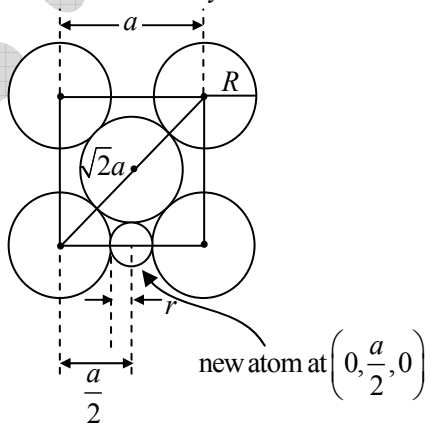


figure (*ii*)

If new atom of radius r fit without distorting the original lattice, then from figure (ii) we get

$$\frac{a}{2} = R + r \qquad \text{(i)}$$

whereas for FCC $\sqrt{2}a = 4R \Rightarrow a = \frac{4}{\sqrt{2}} R = 2\sqrt{2}R$ (ii)

Thus from (i) and (ii) $\frac{2\sqrt{2}}{2}R = R + r \Rightarrow \left(\sqrt{2}-1\right)R = r \Rightarrow \frac{r}{R} = \sqrt{2}-1 = 1.414-1 = 0.414$

Q41. The energy vs. wave vector $(E-k)$ relationship near the bottom of a band for a solid can be approximated as $E = A(ka)^2 + B(ka)^4$, where the lattice constant $a = 2.1\overset{0}{A}$. The values of A and B are $6.3\times10^{-19}\ J$ and $3.2\times10^{-20}\ J$, respectively. At the bottom of the conduction band, the ratio of the effective mass of the electron to the mass of free electron is _____. (Give your answer upto two decimal places)

(Take $\hbar = 1.05\times10^{-34}\ J-s$, mass of free electron $= 9.1\times10^{-31}\ kg$)

Ans.: 0.22

Solution: $E = A(ka)^2 + B(ka)^4$

$\therefore \frac{\partial E}{\partial k} = 2Aa^2k + 4Ba^4k^3$ and $\frac{\partial^2 E}{\partial k^2} = 2Aa^2 + 12Ba^4k^2$

At the bottom of the band $k = 0$

Thus effective mass $m^* = \frac{\hbar^2}{\partial^2 E/\partial k^2} = \frac{\hbar^2}{2Aa^2} = \frac{\left(1.05\times10^{-34}J\cdot s\right)^2}{2\times6.3\times10^{-19}J\times\left(2.1\times10^{-10}m\right)^2}$

$= \frac{1.1025\times10^{-68}}{55.57\times10^{-39}} = 0.01984\times10^{-29} = 19.84\times10^{-32}kg$

$\therefore \frac{m^*}{m} = \frac{19.84\times10^{-32}kg}{9.1\times10^{-31}kg} = 2.18\times10^{-1} = 0.218 \cong 0.22$

Q42. The Fermi energies of two metals X and Y are $5\ eV$ and $7\ eV$ and their Debye temperatures are $170\ K$ and $340\ K$, respectively. The molar specific heats of these metals at constant volume at low temperatures can be written as $(C_V)_X = \gamma_X T + A_X T^3$ and $(C_V)_Y = \gamma_Y T + A_Y T^3$ where γ and A are constants. Assuming that the thermal effective mass of the electrons in the two metals are same, which of the following is correct?

(a) $\frac{\gamma_X}{\gamma_Y} = \frac{7}{5}, \frac{A_X}{A_Y} = 8$

(b) $\frac{\gamma_X}{\gamma_Y} = \frac{7}{5}, \frac{A_X}{A_Y} = \frac{1}{8}$

(c) $\frac{\gamma_X}{\gamma_Y} = \frac{5}{7}, \frac{A_X}{A_Y} = \frac{1}{8}$

(d) $\frac{\gamma_X}{\gamma_Y} = \frac{5}{7}, \frac{A_X}{A_Y} = 8$

Ans.: (a)

Solution: Heat capacity is defined as $C_V = \gamma T + AT^3$

where $\gamma = \frac{3}{2} Nk_B^2 \cdot \frac{1}{E_F}$ and $A = \frac{12\pi^4}{5} Nk_B \cdot \frac{1}{\theta_D^3} \cong 234 Nk_B \cdot \frac{1}{\theta_D^3}$

Thus, $\frac{\gamma_X}{\gamma_Y} = \frac{\frac{3}{2} Nk_B^2 \cdot \frac{1}{E_{F_X}}}{\frac{3}{2} Nk_B^2 \cdot \frac{1}{E_{F_Y}}} = \frac{E_{F_Y}}{E_{F_X}} = \frac{7eV}{5eV} = \frac{7}{5}$

and $\frac{A_X}{A_Y} = \frac{234 Nk_B \frac{1}{\theta_{D_X}^3}}{234 Nk_B \frac{1}{\theta_{D_Y}^3}} = \left(\frac{\theta_{D_Y}}{\theta_{D_X}}\right)^3 = \left(\frac{340}{170}\right)^3 = (2)^3 = 8$

Thus, $\frac{\gamma_X}{\gamma_Y} = \frac{7}{5}$ and $\frac{A_X}{A_Y} = 8$

GATE-2017

Q43. The atomic mass and mass density of Sodium are 23 and $0.968\, g\, cm^{-3}$, respectively. The number density of valence electrons is..... $\times 10^{22}\, cm^{-3}$. (Up to two decimal places) (Avogadro number, $N_A = 6.022 \times 10^{23}$)

Ans. : 2.54

Solution: $\rho = \frac{n_{eff} \times M}{N_A \times a^3} \Rightarrow n = \frac{n_{eff}}{a^3} = \frac{\rho N_A}{M}$

where $\rho = 0.968 g cm^{-3}$, $N_A = 6.022 \times 10^{23}$, $M = 23g$

$$n = \frac{0.968 \times 6.022 \times 10^{23}}{23} = 2.54 \times 10^{22} cm^{-3}$$

Q44. Consider a one dimensional lattice with a weak periodic potential $U(x) = U_0 \cos\left(\frac{2\pi x}{a}\right)$.

The gap at the edge of the Brillouin zone $\left(k = \frac{\pi}{a}\right)$ is:

(a) U_0 (b) $\frac{U_0}{2}$ (c) $2U_0$ (d) $\frac{U_0}{4}$

Ans. : (c)

Solution: $U = U_0 \cos\left(\frac{2\pi}{a}x\right)$

Energy at the edge of Brillouin Zone is $U_t = U_0 \cos\left(\frac{2\pi}{a}.\frac{a}{\pi}\right)$

Energy at the $k = 0$ is $U_b = U_0$ $\quad\therefore$ Band gap $\Delta U = U_t - U_b = -2U_0$

Q45. Consider a 2 - dimensional electron gas with a density of $10^{19}\,m^{-2}$. The Fermi energy of the system is……eV (up to two decimal places).

($m_e = 9.31\times10^{-31}\,kg$, $h = 6.626\times10^{-34}\,Js$, $e = 1.602\times10^{-19}\,C$)

Ans. : 2.34

Solution: $E_F = \left(\frac{\hbar^2}{2m}\right)(2\pi n) = \frac{\left(1.055\times10^{-34}\,J\cdot s\right)^2}{2\times9.31\times10^{-31}}\times2\times3.142\times10^{19}$

$= 0.3756\times10^{-18}\,J = 0.2345\times10\,eV = 2.34\ eV$

Q46. The real space primitive lattice vectors are $\vec{a}_1 = a\hat{x}$ and $\vec{a}_2 = \frac{a}{2}\left(\hat{x}+\sqrt{3}\hat{y}\right)$. The reciprocal space unit vectors $\vec{b}_1$ and $\vec{b}_2$ for this lattice are, respectively

(a) $\frac{2\pi}{a}\left(\hat{x}-\frac{\hat{y}}{\sqrt{3}}\right)$ and $\frac{4\pi}{a\sqrt{3}}\hat{y}$ (b) $\frac{2\pi}{a}\left(\hat{x}+\frac{\hat{y}}{\sqrt{3}}\right)$ and $\frac{4\pi}{a\sqrt{3}}\hat{y}$

(c) $\frac{2\pi}{a\sqrt{3}}\hat{x}$ and $\frac{4\pi}{a}\left(\frac{\hat{x}}{\sqrt{3}}+\hat{y}\right)$ (d) $\frac{2\pi}{a\sqrt{3}}\hat{x}$ and $\frac{4\pi}{a}\left(\frac{\hat{x}}{\sqrt{3}}-\hat{y}\right)$

Ans. (a)

Solution: $\vec{a}_1 = a\hat{x}$, $\vec{a}_2 = \frac{a}{2}\left(\hat{x}+\sqrt{3}\hat{y}\right)$. Assume, $\vec{a}_3 = \hat{z}$

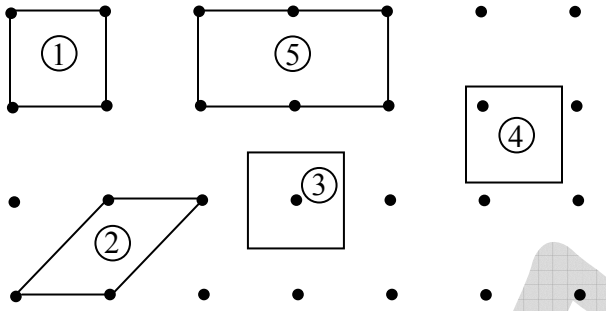
Now, $\vec{a}_1\cdot\left(\vec{a}_2\times\vec{a}_3\right) = a\hat{x}.\left[\frac{a}{2}\left(\hat{x}+\sqrt{3}\hat{y}\right)\times\hat{z}\right] = \frac{a^2\hat{x}}{2}\cdot\left(-\hat{y}+\sqrt{3}\hat{x}\right) = \frac{a^2}{2}\left[-0+\sqrt{3}\right] = \frac{\sqrt{3}a^2}{2}$

$$\therefore \vec{b}_1 = 2\pi\frac{\vec{a}_2\times\vec{a}_3}{\vec{a}_1\cdot\left(\vec{a}_2\times\vec{a}_3\right)} = 2\pi\frac{\left(\sqrt{3}\hat{x}-\hat{y}\right)\frac{a}{2}}{\frac{\sqrt{3}}{2}a^2} = \frac{2\pi}{a}\left(\hat{x}-\frac{\hat{y}}{\sqrt{3}}\right)$$

Similarly, $\vec{b}_2 = 2\pi\frac{\vec{a}_3\times\vec{a}_1}{\vec{a}_1\cdot\left(\vec{a}_2\times\vec{a}_3\right)} = \frac{4\pi}{\sqrt{3}a}\hat{y}$. Thus correct option is (a)

GATE – 2018

Q47. For the given unit cells of a two dimensional square lattice, which option lists all the primitive cells?



(a) (1) and (2)
(b) (1), (2) and (3)
(c) (1), (2), (3) and (4)
(d) (1), (2), (3), (4) and (5)

Ans. : (c)

Solution: For primitive cell, $N_{eff} = 1$

In cell (1), (2), (3) and (4) $N_{eff} = 1$, these are primitive cell

Whereas in cell (5), $N_{eff} = 2$, this is non-primitive cell.

Q48. At low temperatures (T), the specific heat of common metals is described by (with α and β as constants)

(a) $\alpha T + \beta T^3$ (b) βT^3 (c) $\exp(-\alpha / T)$ (d) $\alpha T + \beta T^5$

Ans. : (a)

Solution: $C = C_e + C_{pn} = \alpha T + \beta T^3$

Q49. The high temperature magnetic susceptibility of solids having ions with magnetic moments can be described by $\chi \propto \frac{1}{T+\theta}$ with T as absolute temperature and θ as constant. The three behaviours i.e., paramagnetic, ferromagnetic and anti-ferromagnetic are described, respectively, by

(a) $\theta < 0,\ \theta > 0,\ \theta = 0$
(b) $\theta > 0,\ \theta < 0,\ \theta = 0$
(c) $\theta = 0,\ \theta < 0,\ \theta > 0$
(d) $\theta = 0,\ \theta > 0,\ \theta < 0$

Ans. : (c)

Solution: Paramagnetism: $\chi = \frac{C}{T}$

Ferromagnetism: $\chi = \frac{C}{T - T_C}$

Anti-ferromagnetism: $\chi = \frac{C}{T + T_C}$

Q50. The energy dispersion for electrons in one dimensional lattice with lattice parameter a is given by $E(k) = E_0 - \frac{1}{2}W\cos ka$, where W and E_0 are constants. The effective mass of the electron near the bottom of the band is

(a) $\frac{2\hbar^2}{Wa^2}$ (b) $\frac{\hbar^2}{Wa^2}$ (c) $\frac{\hbar^2}{2Wa^2}$ (d) $\frac{\hbar^2}{4Wa^2}$

Ans. : (a)

Solution: $E(k) = E_0 - \frac{1}{2}W\cos(ka)$

$$\frac{dE}{dk} = \frac{aW}{2}\sin(ka) \Rightarrow \frac{d^2E}{dk^2} = \frac{a^2W}{2}\cos(ka)$$

$$\therefore\ m^* = \frac{\hbar^2}{\frac{d^2E}{dk^2}} = \frac{\hbar^2}{\frac{a^2W}{2}\cos(ka)} = \frac{2\hbar^2}{Wa^2}$$ [At bottom of the band, $k = 0$]

Q51. Amongst electrical resistivity (ρ), thermal conductivity (κ), specific heat (C), Young's modulus (Y) and magnetic susceptibility (χ), which quantities show a sharp change at the superconducting transition temperature?

(a) ρ, κ, C, Y (b) ρ, C, χ (c) ρ, κ, C, χ (d) κ, Y, χ

Ans. : (b)

GATE-2019

Q52. The relative magnetic permeability of a type-I super conductor is

(a) 0 (b) -1 (c) 2π (d) $\frac{1}{4\pi}$

Ans.: (a)

Solution: $B=\mu_0\left(H+M\right)=\mu_0\left(H+\chi H\right)=\mu_0\left(1+\chi\right)H=\mu H$

$$\therefore \quad \mu=\mu_0\left(1+x\right)\Rightarrow \mu_r=\frac{\mu}{\mu_0}=1+\chi$$

For type-I superconductor: $\chi=-1$

$$\therefore \mu_r=1-1=0$$

Q53. In order to estimate the specific heat of phonons, the appropriate method to apply would be

(a) Einstein model for acoustic phonons and Debye model for optical phonons

(b) Einstein model for optical phonons and Debye model for acoustic phonons

(c) Einstein model for both optical and acoustic phonons

(d) Debye model for both optical and acoustic phonons

Ans.: (b)

Solution: At low temperature, the optical branch phonons have energies higher than k_BT and therefore, optical branch waves are not excited. And Debye model is not suitable for optical branch instead it is suitable for acoustical branch. Whereas Einstein model is useful for high temperature and therefore can be applied to optical branch.

Q54. Consider a three-dimensional crystal of N inert gas atoms. The total energy is given by $U\left(R\right)=2N\in\left[p\left(\frac{\sigma}{R}\right)^{12}-q\left(\frac{\sigma}{R}\right)^{6}\right]$, where $p=12.13$, $q=14.45$ and R is the nearest neighbour distance between two atoms. The two constants, $\in$ and R, have the dimensions of energy and length, respectively. The equilibrium separation between two nearest neighbour atoms in units of σ (rounded off to two decimal places) is_____

Ans.: 1.09

Solution: $U(R)=2N\in\left[p\left(\frac{\sigma}{R}\right)^{12}-q\left(\frac{\sigma}{R}\right)^{6}\right]$

$$\frac{dU}{dR}=0\Rightarrow 2N\in\left[12p\left(\frac{\sigma}{R}\right)^{11}\cdot\left(\frac{-\sigma}{R^2}\right)-6q\left(\frac{\sigma}{R}\right)^{5}\left(-\frac{\sigma}{R^2}\right)\right]=0$$

$$\Rightarrow 12p\frac{\sigma^{12}}{R^{13}}-6q\frac{\sigma^{6}}{R^{7}}=0$$

$$\Rightarrow 12p\frac{\sigma^{12}}{R^{13}}=6q\frac{\sigma^{6}}{R^{7}}\Rightarrow R^6=\frac{12p}{6q}\sigma^6$$

$$\Rightarrow R=\left(\frac{2p}{q}\right)^{1/6}\sigma \qquad \text{given } p=12.13,\ q=14.45$$

$$\therefore R=\left(\frac{2\times 12.13}{14.45}\right)^{1/6}\sigma=(1.679)^{1/6}\sigma=1.09\sigma$$

Thus $\frac{R}{\sigma}=1.09$

Q55. The energy-wavevector $(E-k)$ dispersion relation for a particle in two dimensions is $E=Ck$, where C is a constant. If its density of states $D(E)$ is proportional to E^p then the value of p is_____

Ans.: 1

Solution: For $E(k)\propto k^s$. The density of states in d - dimension is $D(E)\propto E^{\left(\frac{d}{s}-1\right)}$

Given, $E=Ck$ $\quad\therefore\quad s=1, d=2$

Thus $D(E)\propto E^{\left(\frac{2}{1}-1\right)}$

$\propto E^1$

Q56. A conventional type-I superconductor has a critical temperature of $4.7\,K$ at zero magnetic field and a critical magnetic field of 0.3 Tesla at $0\,K$. The critical field in Tesla at $2K$ (rounded off to three decimal places) is_____

Ans.: 0.246

Solution: $H_c(T) = H_0\left[1-\left(\frac{T}{T_c}\right)^2\right] = 0.3\left[1-\left(\frac{2}{4.7}\right)^2\right] = 0.3\left[1-(0.426)^2\right]$

$= 0.3[1-0.181] = 0.3\times 0.819 = 0.246\, Atm$

Q57. A particle of mass m moves in a lattice along the x - axis in a periodic potential $V(x) = V(x+d)$ with periodicity d. The corresponding Brillouin zone extends from $-k_0$ to k_0 with these two k - points being equivalent. If a weak force F in the x - direction is applied to the particle, it starts a periodic motion with the time period T. Using the equation of motion $F = \frac{dp_{\text{crystal}}}{dt}$ for a particle moving in a band, where p_{crystal} is the crystal momentum of the particle, the period T is found to be (h is Planck constant)

(a) $\sqrt{\frac{2md}{F}}$ (b) $2\sqrt{\frac{2md}{F}}$ (c) $\frac{2h}{Fd}$ (d) $\frac{h}{Fd}$

Ans. : (d)

Solution: $\Delta E = E\int_0^d Fdx = F[x]_0^d = Fd$

Using Heisenberg uncertainty $\Delta E\cdot\Delta t = h$, $T = \Delta t = \frac{h}{\Delta E} = \frac{h}{Fd}$. Thus correct option is (d)

Q58. In a certain two-dimensional lattice, the energy dispersion of the electrons is

$$\varepsilon(\vec{k}) = -2t\left[\cos k_x a + 2\cos\frac{1}{2}k_x a\cos\frac{\sqrt{3}}{2}k_y a\right]$$

where $\vec{k} = (k_x, k_y)$ denotes the wave vector, a is the lattice constant and t is a constant in units of eV. In this lattice the effective mass tensor m_{ij} of electrons calculated at the center of the Brillouin zone has the form $m_{ij} = \frac{\hbar^2}{ta^2}\begin{pmatrix}\alpha & 0\\ 0 & \alpha\end{pmatrix}$. The value of α (rounded off to two decimal places) is _____

Ans.: 0.333

Solution: Effective mass tensor matrix 4

$$m_{ij} = \begin{bmatrix} \frac{1}{m_{xx}} & \frac{1}{m_{xy}} \\ \frac{1}{m_{yx}} & \frac{1}{m_{yy}} \end{bmatrix} = \begin{bmatrix} \frac{1}{m_{xx}} & 0 \\ 0 & \frac{1}{m_{yy}} \end{bmatrix}$$

When $m_{xx} = \dfrac{\hbar^2}{\partial^2 E / \partial k_x^2}$ and $m_{yy} = \dfrac{\hbar^2}{\partial^2 E / \partial k_y^2}$

Now $\dfrac{\partial E}{\partial k_x} = 2t\left[a\sin k_x a + a\sin\left(\frac{1}{2}k_x a\right)\cos\left(\frac{\sqrt{3}}{2}k_y a\right)\right]$

$$\frac{\partial^2 E}{dk_x^2} = 2t\left[a^2\cos\left(k_x a\right) + \frac{a^2}{2}\cos\left(\frac{1}{2}k_x a\right)\cos\left(\frac{\sqrt{3}}{2}k_y a\right)\right]$$

At the Brillouin zone centre i.e. at $k_x = k_y = 0$

$$\therefore \qquad \frac{\partial^2 E}{\partial k_x^2} = 2ta^2\left(1+\frac{1}{2}\right) = 3ta^2$$

Similarly, $\dfrac{\partial E}{\partial k_y} = 2t\left[\sqrt{3}a\cos\left(\frac{1}{2}k_x a\right)\sin\left(\frac{\sqrt{3}}{2}k_y a\right)\right]$

$$\frac{\partial^2 E}{\partial k_y^2} = 2t\left[\frac{3a^2}{2}\cos\left(\frac{1}{2}k_x a\right)\cos\left(\frac{\sqrt{3}}{2}k_y a\right)\right]$$

At the Brillouin zone centre i.e. at $k_x = k_y = 0$

$$\frac{\partial^2 E}{\partial_y^2} = 3ta^2$$

Thus $m_{xx} = \dfrac{\hbar^2}{\partial^2 E / \partial k_x^2} = \dfrac{\hbar^2}{3ta^2}$ and $m_{yy} = \dfrac{\hbar^2}{\partial^2 E / \partial k_y^2} = \dfrac{\hbar^2}{3ta^2}$

$$m_{ij} = \begin{bmatrix} \frac{\hbar^2}{3ta^2} & 0 \\ 0 & \frac{\hbar^2}{3ta^2} \end{bmatrix} = \frac{\hbar^2}{ta^2}\begin{bmatrix} \frac{1}{3} & 0 \\ 0 & \frac{1}{3} \end{bmatrix}$$

Thus $\alpha = \dfrac{1}{3} = 0.333$.